11 | THE CHI-SQUARE DISTRIBUTION

Figure 11.1 The chi-square distribution can be used to find relationships between two things, like grocery prices at different stores. (credit: Pete/flickr)

Introduction

Chapter Objectives

By the end of this chapter, the student should be able to:

- Interpret the chi-square probability distribution as the sample size changes.
- Conduct and interpret chi-square goodness-of-fit hypothesis tests.
- Conduct and interpret chi-square test of independence hypothesis tests.
- Conduct and interpret chi-square homogeneity hypothesis tests.
- Conduct and interpret chi-square single variance hypothesis tests.

Have you ever wondered if lottery numbers were evenly distributed or if some numbers occurred with a greater frequency? How about if the types of movies people preferred were different across different age groups? What about if a coffee machine was dispensing approximately the same amount of coffee each time? You could answer these questions by conducting a hypothesis test.

You will now study a new distribution, one that is used to determine the answers to such questions. This distribution is called the chi-square distribution.

In this chapter, you will learn the three major applications of the chi-square distribution:

1. the goodness-of-fit test, which determines if data fit a particular distribution, such as in the lottery example
2. the test of independence, which determines if events are independent, such as in the movie example
3. the test of a single variance, which tests variability, such as in the coffee example

NOTE

Though the chi-square distribution depends on calculators or computers for most of the calculations, there is a table available (see **Appendix G**). TI-83+ and TI-84 calculator instructions are included in the text.

Collaborative Exercise

Look in the sports section of a newspaper or on the Internet for some sports data (baseball averages, basketball scores, golf tournament scores, football odds, swimming times, and the like). Plot a histogram and a boxplot using your data. See if you can determine a probability distribution that your data fits. Have a discussion with the class about your choice.

11.1 | Facts About the Chi-Square Distribution

The notation for the **chi-square distribution** is:

$$\chi \sim \chi^2_{df}$$

where *df* = degrees of freedom which depends on how chi-square is being used. (If you want to practice calculating chi-square probabilities then use *df* = *n* - 1. The degrees of freedom for the three major uses are each calculated differently.)

For the χ^2 distribution, the population mean is $\mu = df$ and the population standard deviation is $\sigma = \sqrt{2(df)}$.

The random variable is shown as χ^2, but may be any upper case letter.

The random variable for a chi-square distribution with *k* degrees of freedom is the sum of *k* independent, squared standard normal variables.

$\chi^2 = (Z_1)^2 + (Z_2)^2 + ... + (Z_k)^2$

1. The curve is nonsymmetrical and skewed to the right.
2. There is a different chi-square curve for each *df*.

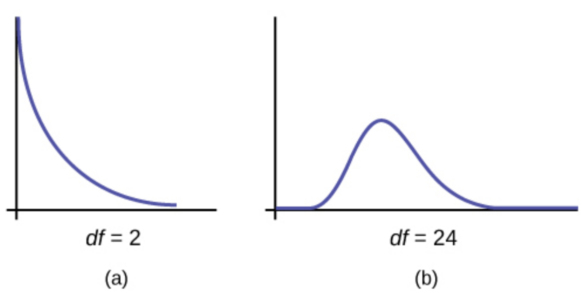

Figure 11.2

3. The test statistic for any test is always greater than or equal to zero.

4. When $df > 90$, the chi-square curve approximates the normal distribution. For $X \sim \chi^2_{1,000}$ the mean, $\mu = df = 1{,}000$ and the standard deviation, $\sigma = \sqrt{2(1{,}000)} = 44.7$. Therefore, $X \sim N(1{,}000, 44.7)$, approximately.
5. The mean, μ, is located just to the right of the peak.

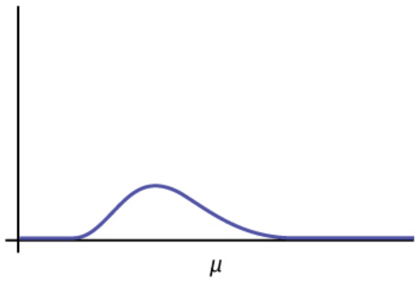

Figure 11.3

11.2 | Goodness-of-Fit Test

In this type of hypothesis test, you determine whether the data **"fit"** a particular distribution or not. For example, you may suspect your unknown data fit a binomial distribution. You use a chi-square test (meaning the distribution for the hypothesis test is chi-square) to determine if there is a fit or not. **The null and the alternative hypotheses for this test may be written in sentences or may be stated as equations or inequalities.**

The test statistic for a goodness-of-fit test is:

$$\sum_{k} \frac{(O - E)^2}{E}$$

where:

- O = **observed values** (data)
- E = **expected values** (from theory)
- k = the number of different data cells or categories

The observed values are the data values and the expected values are the values you would expect to get if the null hypothesis were true. There are n terms of the form $\frac{(O - E)^2}{E}$.

The number of degrees of freedom is df = (number of categories – 1).

The goodness-of-fit test is almost always right-tailed. If the observed values and the corresponding expected values are not close to each other, then the test statistic can get very large and will be way out in the right tail of the chi-square curve.

NOTE

The expected value for each cell needs to be at least five in order for you to use this test.

Example 11.1

Absenteeism of college students from math classes is a major concern to math instructors because missing class appears to increase the drop rate. Suppose that a study was done to determine if the actual student absenteeism rate follows faculty perception. The faculty expected that a group of 100 students would miss class according to **Table 11.1**.

Number of absences per term	Expected number of students
0–2	50
3–5	30
6–8	12
9–11	6
12+	2

Table 11.1

A random survey across all mathematics courses was then done to determine the actual number **(observed)** of absences in a course. The chart in Table 11.2 displays the results of that survey.

Number of absences per term	Actual number of students
0–2	35
3–5	40
6–8	20
9–11	1
12+	4

Table 11.2

Determine the null and alternative hypotheses needed to conduct a goodness-of-fit test.

***H_0*: Student absenteeism **fits** faculty perception.

The alternative hypothesis is the opposite of the null hypothesis.

***H_a*: Student absenteeism **does not fit** faculty perception.

a. Can you use the information as it appears in the charts to conduct the goodness-of-fit test?

Solution 11.1

a. **No.** Notice that the expected number of absences for the "12+" entry is less than five (it is two). Combine that group with the "9–11" group to create new tables where the number of students for each entry are at least five. The new results are in Table 11.2 and Table 11.3.

Number of absences per term	Expected number of students
0–2	50
3–5	30
6–8	12
9+	8

Table 11.3

Number of absences per term	Actual number of students
0–2	35

Number of absences per term	Actual number of students
3–5	40
6–8	20
9+	5

Table 11.4

b. What is the number of degrees of freedom (*df*)?

Solution 11.1

b. There are four "cells" or categories in each of the new tables.

df = number of cells – 1 = 4 – 1 = 3

Try It Σ

11.1 A factory manager needs to understand how many products are defective versus how many are produced. The number of expected defects is listed in Table 11.5.

Number produced	Number defective
0–100	5
101–200	6
201–300	7
301–400	8
401–500	10

Table 11.5

A random sample was taken to determine the actual number of defects. Table 11.6 shows the results of the survey.

Number produced	Number defective
0–100	5
101–200	7
201–300	8
301–400	9
401–500	11

Table 11.6

State the null and alternative hypotheses needed to conduct a goodness-of-fit test, and state the degrees of freedom.

Example 11.2

Employers want to know which days of the week employees are absent in a five-day work week. Most employers would like to believe that employees are absent equally during the week. Suppose a random sample of 60 managers were asked on which day of the week they had the highest number of employee absences. The results were distributed as in **Table 11.6**. For the population of employees, do the days for the highest number of absences occur with equal frequencies during a five-day work week? Test at a 5% significance level.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Number of Absences	15	12	9	9	15

Table 11.7 Day of the Week Employees were Most Absent

Solution 11.2

The null and alternative hypotheses are:

- H_0: The absent days occur with equal frequencies, that is, they fit a uniform distribution.
- H_a: The absent days occur with unequal frequencies, that is, they do not fit a uniform distribution.

If the absent days occur with equal frequencies, then, out of 60 absent days (the total in the sample: 15 + 12 + 9 + 9 + 15 = 60), there would be 12 absences on Monday, 12 on Tuesday, 12 on Wednesday, 12 on Thursday, and 12 on Friday. These numbers are the **expected** (*E*) values. The values in the table are the **observed** (*O*) values or data.

This time, calculate the χ^2 test statistic by hand. Make a chart with the following headings and fill in the columns:

- Expected (*E*) values (12, 12, 12, 12, 12)
- Observed (*O*) values (15, 12, 9, 9, 15)
- $(O - E)$
- $(O - E)^2$
- $\frac{(O-E)^2}{E}$

Now add (sum) the last column. The sum is three. This is the χ^2 test statistic.

To find the *p*-value, calculate $P(\chi^2 > 3)$. This test is right-tailed. (Use a computer or calculator to find the *p*-value. You should get *p*-value = 0.5578.)

The *dfs* are the number of cells – 1 = 5 – 1 = 4

Using the TI-83, 83+, 84, 84+ Calculator

Press `2nd DISTR`. Arrow down to `χ2cdf`. Press `ENTER`. Enter `(3,10^99,4)`. Rounded to four decimal places, you should see 0.5578, which is the p-value.

Next, complete a graph like the following one with the proper labeling and shading. (You should shade the right tail.)

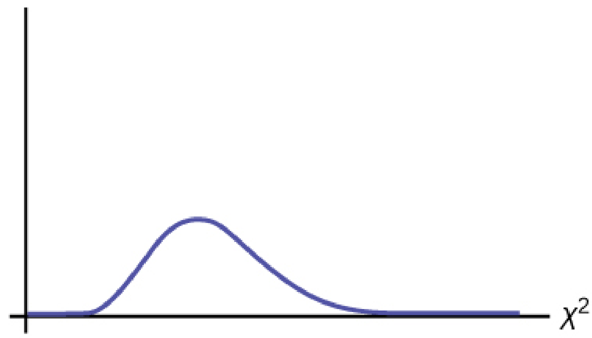

Figure 11.4

The decision is not to reject the null hypothesis.

Conclusion: At a 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the absent days do not occur with equal frequencies.

Using the TI-83, 83+, 84, 84+ Calculator

TI-83+ and some TI-84 calculators do not have a special program for the test statistic for the goodness-of-fit test. The next example **Example 11.3** has the calculator instructions. The newer TI-84 calculators have in `STAT TESTS` the test `Chi2 GOF`. To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true) into a second list. Press `STAT TESTS` and `Chi2 GOF`. Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press `calculate` or `draw`. Make sure you clear any lists before you start. **To Clear Lists in the calculators:** Go into `STAT EDIT` and arrow up to the list name area of the particular list. Press `CLEAR` and then arrow down. The list will be cleared. Alternatively, you can press `STAT` and press 4 (for `ClrList`). Enter the list name and press `ENTER`.

Try It Σ

11.2 Teachers want to know which night each week their students are doing most of their homework. Most teachers think that students do homework equally throughout the week. Suppose a random sample of 49 students were asked on which night of the week they did the most homework. The results were distributed as in **Table 11.8**.

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Students	11	8	10	7	10	5	5

Table 11.8

From the population of students, do the nights for the highest number of students doing the majority of their homework occur with equal frequencies during a week? What type of hypothesis test should you use?

Example 11.3

One study indicates that the number of televisions that American families have is distributed (this is the **given** distribution for the American population) as in **Table 11.9**.

Number of Televisions	Percent
0	10
1	16
2	55
3	11
4+	8

Table 11.9

The table contains expected (*E*) percents.

A random sample of 600 families in the far western United States resulted in the data in **Table 11.10**.

Number of Televisions	Frequency
0	66
1	119
2	340
3	60
4+	15
	Total = 600

Table 11.10

The table contains observed (*O*) frequency values.

At the 1% significance level, does it appear that the distribution "number of televisions" of far western United States families is different from the distribution for the American population as a whole?

Solution 11.3

This problem asks you to test whether the far western United States families distribution fits the distribution of the American families. This test is always right-tailed.

The first table contains expected percentages. To get expected (*E*) frequencies, multiply the percentage by 600. The expected frequencies are shown in **Table 11.10**.

Number of Televisions	Percent	Expected Frequency
0	10	(0.10)(600) = 60
1	16	(0.16)(600) = 96
2	55	(0.55)(600) = 330
3	11	(0.11)(600) = 66
over 3	8	(0.08)(600) = 48

Table 11.11

Therefore, the expected frequencies are 60, 96, 330, 66, and 48. In the TI calculators, you can let the calculator do the math. For example, instead of 60, enter 0.10*600.

H_0: The "number of televisions" distribution of far western United States families is the same as the "number of televisions" distribution of the American population.

H_a: The "number of televisions" distribution of far western United States families is different from the "number of televisions" distribution of the American population.

Distribution for the test: χ_4^2 where *df* = (the number of cells) – 1 = 5 – 1 = 4.

NOTE

df ≠ 600 – 1

Calculate the test statistic: $\chi 2$ = 29.65

Graph:

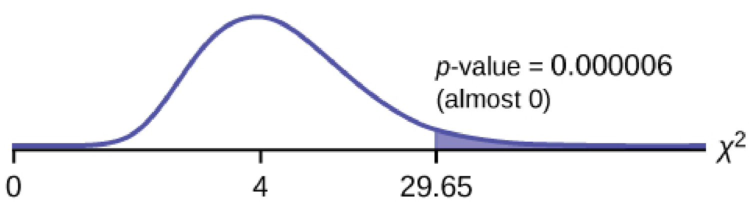

Figure 11.5

Probability statement: *p*-value = $P(\chi^2 > 29.65)$ = 0.000006

Compare *α* and the *p*-value:

- α = 0.01
- *p*-value = 0.000006

So, α > *p*-value.

Make a decision: Since α > *p*-value, reject H_o.

This means you reject the belief that the distribution for the far western states is the same as that of the American population as a whole.

Conclusion: At the 1% significance level, from the data, there is sufficient evidence to conclude that the "number of televisions" distribution for the far western United States is different from the "number of televisions" distribution for the American population as a whole.

Using the TI-83, 83+, 84, 84+ Calculator

Press `STAT` and `ENTER`. Make sure to clear lists `L1`, `L2`, and `L3` if they have data in them (see the note at the end of **Example 11.2**). Into `L1`, put the observed frequencies `66`, `119`, `349`, `60`, `15`. Into `L2`, put the expected frequencies `.10*600, .16*600`, `.55*600`, `.11*600`, `.08*600`. Arrow over to list `L3` and up to the name area `"L3"`. Enter `(L1-L2)^2/L2` and `ENTER`. Press `2nd QUIT`. Press `2nd LIST` and arrow over to `MATH`. Press `5`. You should see `"sum" (Enter L3)`. Rounded to 2 decimal places, you should see `29.65`. Press `2nd DISTR`. Press `7` or Arrow down to `7:χ2cdf` and press `ENTER`. Enter `(29.65,1E99,4)`. Rounded to four places, you should see `5.77E-6 = .000006` (rounded to six decimal places), which is the p-value.

The newer TI-84 calculators have in `STAT TESTS` the test `Chi2 GOF`. To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true) into a second list. Press `STAT TESTS` and `Chi2 GOF`. Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press `calculate` or `draw`. Make sure you clear any lists before you start.

Try It Σ

11.3 The expected percentage of the number of pets students have in their homes is distributed (this is the given distribution for the student population of the United States) as in **Table 11.12**.

Number of Pets	Percent
0	18
1	25
2	30
3	18
4+	9

Table 11.12

A random sample of 1,000 students from the Eastern United States resulted in the data in **Table 11.13**.

Number of Pets	Frequency
0	210
1	240
2	320
3	140
4+	90

Table 11.13

At the 1% significance level, does it appear that the distribution “number of pets” of students in the Eastern United States is different from the distribution for the United States student population as a whole? What is the *p*-value?

Example 11.4

Suppose you flip two coins 100 times. The results are 20 *HH*, 27 *HT*, 30 *TH*, and 23 *TT*. Are the coins fair? Test at a 5% significance level.

Solution 11.4

This problem can be set up as a goodness-of-fit problem. The sample space for flipping two fair coins is {*HH*, *HT*, *TH*, *TT*}. Out of 100 flips, you would expect 25 *HH*, 25 *HT*, 25 *TH*, and 25 *TT*. This is the expected distribution. The question, "Are the coins fair?" is the same as saying, "Does the distribution of the coins (20 *HH*, 27 *HT*, 30 *TH*, 23 *TT*) fit the expected distribution?"

Random Variable: Let X = the number of heads in one flip of the two coins. X takes on the values 0, 1, 2. (There are 0, 1, or 2 heads in the flip of two coins.) Therefore, the **number of cells is three**. Since X = the number of heads, the observed frequencies are 20 (for two heads), 57 (for one head), and 23 (for zero heads or both tails). The expected frequencies are 25 (for two heads), 50 (for one head), and 25 (for zero heads or both tails). This test is right-tailed.

H_0: The coins are fair.

H_a: The coins are not fair.

Distribution for the test: χ^2_2 where $df = 3 – 1 = 2$.

Calculate the test statistic: $\chi^2 = 2.14$

Graph:

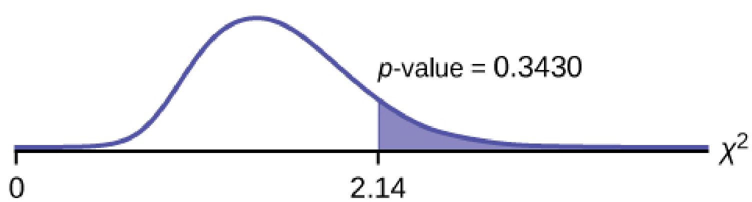

Figure 11.6

Probability statement: p-value = $P(\chi^2 > 2.14) = 0.3430$

Compare α and the p-value:

- $\alpha = 0.05$
- p-value = 0.3430

$\alpha < p$-value.

Make a decision: Since $\alpha < p$-value, do not reject H_0.

Conclusion: There is insufficient evidence to conclude that the coins are not fair.

Using the TI-83, 83+, 84, 84+ Calculator

Press `STAT` and `ENTER`. Make sure you clear lists `L1`, `L2`, and `L3` if they have data in them. Into `L1`, put the observed frequencies `20`, `57`, `23`. Into `L2`, put the expected frequencies `25`, `50`, `25`. Arrow over to list `L3` and up to the name area `"L3"`. Enter `(L1-L2)^2/L2` and `ENTER`. Press `2nd QUIT`. Press `2nd LIST` and arrow over to `MATH`. Press `5`. You should see `"sum".Enter L3`. Rounded to two decimal places, you should see `2.14`. Press `2nd DISTR`. Arrow down to `7:χ2cdf` (or press `7`). Press `ENTER`. Enter `2.14,1E99,2)`. Rounded to four places, you should see `.3430`, which is the p-value.

The newer TI-84 calculators have in `STAT TESTS` the test `Chi2 GOF`. To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true)

into a second list. Press `STAT TESTS` and `Chi2 GOF`. Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press `calculate` or `draw`. Make sure you clear any lists before you start.

Try It Σ

11.4 Students in a social studies class hypothesize that the literacy rates across the world for every region are 82%. **Table 11.14** shows the actual literacy rates across the world broken down by region. What are the test statistic and the degrees of freedom?

MDG Region	Adult Literacy Rate (%)
Developed Regions	99.0
Commonwealth of Independent States	99.5
Northern Africa	67.3
Sub-Saharan Africa	62.5
Latin America and the Caribbean	91.0
Eastern Asia	93.8
Southern Asia	61.9
South-Eastern Asia	91.9
Western Asia	84.5
Oceania	66.4

Table 11.14

11.3 | Test of Independence

Tests of independence involve using a **contingency table** of observed (data) values.

The test statistic for a **test of independence** is similar to that of a goodness-of-fit test:

$$\sum_{(i \cdot j)} \frac{(O - E)^2}{E}$$

where:

- O = observed values
- E = expected values
- i = the number of rows in the table
- j = the number of columns in the table

There are $i \cdot j$ terms of the form $\frac{(O - E)^2}{E}$.

A test of independence determines whether two factors are independent or not. You first encountered the term independence in **Probability Topics**. As a review, consider the following example.

NOTE

The expected value for each cell needs to be at least five in order for you to use this test.

Example 11.5

Suppose A = a speeding violation in the last year and B = a cell phone user while driving. If A and B are independent then $P(A \text{ AND } B) = P(A)P(B)$. A AND B is the event that a driver received a speeding violation last year and also used a cell phone while driving. Suppose, in a study of drivers who received speeding violations in the last year, and who used cell phone while driving, that 755 people were surveyed. Out of the 755, 70 had a speeding violation and 685 did not; 305 used cell phones while driving and 450 did not.

Let y = expected number of drivers who used a cell phone while driving and received speeding violations.

If A and B are independent, then $P(A \text{ AND } B) = P(A)P(B)$. By substitution,

$$\frac{y}{755} = \left(\frac{70}{755}\right)\left(\frac{305}{755}\right)$$

Solve for y: $y = \dfrac{(70)(305)}{755} = 28.3$

About 28 people from the sample are expected to use cell phones while driving and to receive speeding violations.

In a test of independence, we state the null and alternative hypotheses in words. Since the contingency table consists of **two factors**, the null hypothesis states that the factors are **independent** and the alternative hypothesis states that they are **not independent (dependent)**. If we do a test of independence using the example, then the null hypothesis is:

H_0: Being a cell phone user while driving and receiving a speeding violation are independent events.

If the null hypothesis were true, we would expect about 28 people to use cell phones while driving and to receive a speeding violation.

The test of independence is always right-tailed because of the calculation of the test statistic. If the expected and observed values are not close together, then the test statistic is very large and way out in the right tail of the chi-square curve, as it is in a goodness-of-fit.

The number of degrees of freedom for the test of independence is:

df = (number of columns - 1)(number of rows - 1)

The following formula calculates the **expected number** (E):

$$E = \frac{\text{(row total)(column total)}}{\text{total number surveyed}}$$

Try It Σ

11.5 A sample of 300 students is taken. Of the students surveyed, 50 were music students, while 250 were not. Ninety-seven were on the honor roll, while 203 were not. If we assume being a music student and being on the honor roll are independent events, what is the expected number of music students who are also on the honor roll?

Example 11.6

In a volunteer group, adults 21 and older volunteer from one to nine hours each week to spend time with a disabled senior citizen. The program recruits among community college students, four-year college students, and nonstudents. In **Table 11.15** is a **sample** of the adult volunteers and the number of hours they volunteer per week.

Type of Volunteer	1–3 Hours	4–6 Hours	7–9 Hours	Row Total
Community College Students	111	96	48	255
Four-Year College Students	96	133	61	290
Nonstudents	91	150	53	294
Column Total	298	379	162	839

Table 11.15 Number of Hours Worked Per Week by Volunteer Type (Observed) The table contains **observed (O)** values (data).

Is the number of hours volunteered **independent** of the type of volunteer?

Solution 11.6

The **observed table** and the question at the end of the problem, "Is the number of hours volunteered independent of the type of volunteer?" tell you this is a test of independence. The two factors are **number of hours volunteered** and **type of volunteer**. This test is always right-tailed.

H_0: The number of hours volunteered is **independent** of the type of volunteer.

H_a: The number of hours volunteered is **dependent** on the type of volunteer.

The expected result are in Table 11.15.

Type of Volunteer	1-3 Hours	4-6 Hours	7-9 Hours
Community College Students	90.57	115.19	49.24
Four-Year College Students	103.00	131.00	56.00
Nonstudents	104.42	132.81	56.77

Table 11.16 Number of Hours Worked Per Week by Volunteer Type (Expected) The table contains **expected** (*E*) values (data).

For example, the calculation for the expected frequency for the top left cell is

$$E = \frac{(\text{row total})(\text{column total})}{\text{total number surveyed}} = \frac{(255)(298)}{839} = 90.57$$

Calculate the test statistic: χ^2 = 12.99 (calculator or computer)

Distribution for the test: χ^2_4

df = (3 columns – 1)(3 rows – 1) = (2)(2) = 4

Graph:

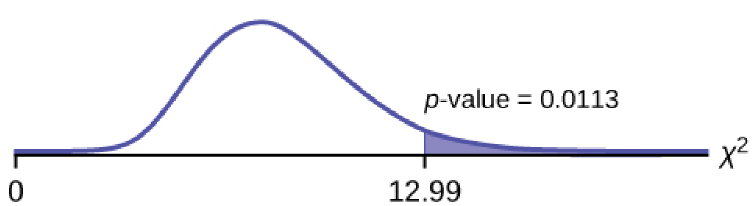

Figure 11.7

Probability statement: p-value=$P(\chi^2 > 12.99) = 0.0113$

Compare α and the p-value: Since no α is given, assume $\alpha = 0.05$. p-value = 0.0113. $\alpha > p$-value.

Make a decision: Since $\alpha > p$-value, reject H_0. This means that the factors are not independent.

Conclusion: At a 5% level of significance, from the data, there is sufficient evidence to conclude that the number of hours volunteered and the type of volunteer are dependent on one another.

For the example in **Table 11.15**, if there had been another type of volunteer, teenagers, what would the degrees of freedom be?

Using the TI-83, 83+, 84, 84+ Calculator

Press the `MATRX` key and arrow over to `EDIT`. Press `1:[A]`. Press `3 ENTER 3 ENTER`. Enter the table values by row from **Table 11.15**. Press `ENTER` after each. Press `2nd QUIT`. Press `STAT` and arrow over to `TESTS`. Arrow down to `C:χ2-TEST`. Press `ENTER`. You should see `Observed:[A] and Expected:[B]`. Arrow down to `Calculate`. Press `ENTER`. The test statistic is 12.9909 and the p-value = 0.0113. Do the procedure a second time, but arrow down to `Draw` instead of `calculate`.

Try It Σ

11.6 The Bureau of Labor Statistics gathers data about employment in the United States. A sample is taken to calculate the number of U.S. citizens working in one of several industry sectors over time. **Table 11.17** shows the results:

Industry Sector	2000	2010	2020	Total
Nonagriculture wage and salary	13,243	13,044	15,018	41,305
Goods-producing, excluding agriculture	2,457	1,771	1,950	6,178
Services-providing	10,786	11,273	13,068	35,127
Agriculture, forestry, fishing, and hunting	240	214	201	655
Nonagriculture self-employed and unpaid family worker	931	894	972	2,797
Secondary wage and salary jobs in agriculture and private household industries	14	11	11	36
Secondary jobs as a self-employed or unpaid family worker	196	144	152	492
Total	27,867	27,351	31,372	86,590

Table 11.17

We want to know if the change in the number of jobs is independent of the change in years. State the null and alternative hypotheses and the degrees of freedom.

Example 11.7

De Anza College is interested in the relationship between anxiety level and the need to succeed in school. A random sample of 400 students took a test that measured anxiety level and need to succeed in school. **Table 11.18** shows the results. De Anza College wants to know if anxiety level and need to succeed in school are independent events.

Need to Succeed in School	High Anxiety	Med-high Anxiety	Medium Anxiety	Med-low Anxiety	Low Anxiety	Row Total
High Need	35	42	53	15	10	155
Medium Need	18	48	63	33	31	193
Low Need	4	5	11	15	17	52
Column Total	57	95	127	63	58	400

Table 11.18 Need to Succeed in School vs. Anxiety Level

a. How many high anxiety level students are expected to have a high need to succeed in school?

Solution 11.7

a. The column total for a high anxiety level is 57. The row total for high need to succeed in school is 155. The sample size or total surveyed is 400.

$$E = \frac{\text{(row total)(column total)}}{\text{total surveyed}} = \frac{155 \cdot 57}{400} = 22.09$$

The expected number of students who have a high anxiety level and a high need to succeed in school is about 22.

b. If the two variables are independent, how many students do you expect to have a low need to succeed in school and a med-low level of anxiety?

Solution 11.7

b. The column total for a med-low anxiety level is 63. The row total for a low need to succeed in school is 52. The sample size or total surveyed is 400.

c. $E = \frac{\text{(row total)(column total)}}{\text{total surveyed}} =$ ________

Solution 11.7

c. $E = \frac{\text{(row total)(column total)}}{\text{total surveyed}} = 8.19$

d. The expected number of students who have a med-low anxiety level and a low need to succeed in school is about ________.

Solution 11.7

d. 8

Try It Σ

11.7 Refer back to the information in **Try It**. How many service providing jobs are there expected to be in 2020? How many nonagriculture wage and salary jobs are there expected to be in 2020?

11.4 | Test for Homogeneity

The goodness–of–fit test can be used to decide whether a population fits a given distribution, but it will not suffice to decide whether two populations follow the same unknown distribution. A different test, called the **test for homogeneity**, can be used to draw a conclusion about whether two populations have the same distribution. To calculate the test statistic for a test for homogeneity, follow the same procedure as with the test of independence.

NOTE

The expected value for each cell needs to be at least five in order for you to use this test.

Hypotheses

H_0: The distributions of the two populations are the same.

H_a: The distributions of the two populations are not the same.

Test Statistic

Use a χ^2 test statistic. It is computed in the same way as the test for independence.

Degrees of Freedom (*df*)

df = number of columns - 1

Requirements

All values in the table must be greater than or equal to five.

Common Uses

Comparing two populations. For example: men vs. women, before vs. after, east vs. west. The variable is categorical with more than two possible response values.

Example 11.8

Do male and female college students have the same distribution of living arrangements? Use a level of significance of 0.05. Suppose that 250 randomly selected male college students and 300 randomly selected female college students were asked about their living arrangements: dormitory, apartment, with parents, other. The results are shown in **Table 11.18**. Do male and female college students have the same distribution of living arrangements?

	Dormitory	Apartment	With Parents	Other
Males	72	84	49	45
Females	91	86	88	35

Table 11.19 Distribution of Living Arragements for College Males and College Females

Solution 11.8

H_0: The distribution of living arrangements for male college students is the same as the distribution of living arrangements for female college students.

H_a: The distribution of living arrangements for male college students is not the same as the distribution of living arrangements for female college students.

Degrees of Freedom (*df*):
df = number of columns – 1 = 4 – 1 = 3

Distribution for the test: χ^2_3

Calculate the test statistic: χ^2 = 10.1287 (calculator or computer)

Probability statement: *p*-value = $P(\chi^2 > 10.1287) = 0.0175$

Using the TI-83, 83+, 84, 84+ Calculator

Press the `MATRX` key and arrow over to `EDIT`. Press `1:[A]`. Press `2 ENTER 4 ENTER`. Enter the table values by row. Press `ENTER` after each. Press `2nd QUIT`. Press `STAT` and arrow over to `TESTS`. Arrow down to `C:χ2-TEST`. Press `ENTER`. You should see `Observed:[A] and Expected:[B]`. Arrow down to `Calculate`. Press `ENTER`. The test statistic is 10.1287 and the *p*-value = 0.0175. Do the procedure a second time but arrow down to `Draw` instead of `calculate`.

Compare α and the *p*-value: Since no α is given, assume $\alpha = 0.05$. *p*-value = 0.0175. $\alpha > p$-value.

Make a decision: Since $\alpha > p$-value, reject H_0. This means that the distributions are not the same.

Conclusion: At a 5% level of significance, from the data, there is sufficient evidence to conclude that the distributions of living arrangements for male and female college students are not the same.

Notice that the conclusion is only that the distributions are not the same. We cannot use the test for homogeneity to draw any conclusions about how they differ.

Try It Σ

11.8 Do families and singles have the same distribution of cars? Use a level of significance of 0.05. Suppose that 100 randomly selected families and 200 randomly selected singles were asked what type of car they drove: sport, sedan, hatchback, truck, van/SUV. The results are shown in **Table 11.20**. Do families and singles have the same distribution of cars? Test at a level of significance of 0.05.

	Sport	Sedan	Hatchback	Truck	Van/SUV
Family	5	15	35	17	28
Single	45	65	37	46	7

Table 11.20

Example 11.9

Both before and after a recent earthquake, surveys were conducted asking voters which of the three candidates they planned on voting for in the upcoming city council election. Has there been a change since the earthquake? Use a level of significance of 0.05. **Table 11.20** shows the results of the survey. Has there been a change in the distribution of voter preferences since the earthquake?

	Perez	Chung	Stevens
Before	167	128	135
After	214	197	225

Table 11.21

Solution 11.9

H_0: The distribution of voter preferences was the same before and after the earthquake.

H_a: The distribution of voter preferences was not the same before and after the earthquake.

Degrees of Freedom (*df*):
df = number of columns – 1 = 3 – 1 = 2

Distribution for the test: χ^2_2

Calculate the test statistic: χ^2 = 3.2603 (calculator or computer)

Probability statement: p-value=$P(\chi^2 > 3.2603) = 0.1959$

Using the TI-83, 83+, 84, 84+ Calculator

Press the `MATRX` key and arrow over to `EDIT`. Press `1:[A]`. Press `2 ENTER 3 ENTER`. Enter the table values by row. Press `ENTER` after each. Press `2nd QUIT`. Press `STAT` and arrow over to `TESTS`. Arrow down to `C:χ2-TEST`. Press `ENTER`. You should see `Observed:[A] and Expected:[B]`. Arrow down to `Calculate`. Press `ENTER`. The test statistic is 3.2603 and the p-value = 0.1959. Do the procedure a second time but arrow down to `Draw` instead of `calculate`.

Compare *α* and the *p*-value: α = 0.05 and the p-value = 0.1959. $\alpha < p$-value.

Make a decision: Since $\alpha < p$-value, do not reject H_o.

Conclusion: At a 5% level of significance, from the data, there is insufficient evidence to conclude that the distribution of voter preferences was not the same before and after the earthquake.

Try It Σ

11.9 Ivy League schools receive many applications, but only some can be accepted. At the schools listed in Table 11.22, two types of applications are accepted: regular and early decision.

Application Type Accepted	Brown	Columbia	Cornell	Dartmouth	Penn	Yale
Regular	2,115	1,792	5,306	1,734	2,685	1,245
Early Decision	577	627	1,228	444	1,195	761

Table 11.22

We want to know if the number of regular applications accepted follows the same distribution as the number of early applications accepted. State the null and alternative hypotheses, the degrees of freedom and the test statistic, sketch the graph of the p-value, and draw a conclusion about the test of homogeneity.

11.5 | Comparison of the Chi-Square Tests

You have seen the χ^2 test statistic used in three different circumstances. The following bulleted list is a summary that will help you decide which χ^2 test is the appropriate one to use.

- **Goodness-of-Fit:** Use the goodness-of-fit test to decide whether a population with an unknown distribution "fits" a known distribution. In this case there will be a single qualitative survey question or a single outcome of an experiment from a single population. Goodness-of-Fit is typically used to see if the population is uniform (all outcomes occur

with equal frequency), the population is normal, or the population is the same as another population with a known distribution. The null and alternative hypotheses are:
H_0: The population fits the given distribution.
H_a: The population does not fit the given distribution.

- **Independence:** Use the test for independence to decide whether two variables (factors) are independent or dependent. In this case there will be two qualitative survey questions or experiments and a contingency table will be constructed. The goal is to see if the two variables are unrelated (independent) or related (dependent). The null and alternative hypotheses are:
H_0: The two variables (factors) are independent.
H_a: The two variables (factors) are dependent.
- **Homogeneity:** Use the test for homogeneity to decide if two populations with unknown distributions have the same distribution as each other. In this case there will be a single qualitative survey question or experiment given to two different populations. The null and alternative hypotheses are:
H_0: The two populations follow the same distribution.
H_a: The two populations have different distributions.

11.6 | Test of a Single Variance

A **test of a single variance** assumes that the underlying distribution is **normal**. The null and alternative hypotheses are stated in terms of the **population variance** (or population standard deviation). The test statistic is:

$$\frac{(n-1)s^2}{\sigma^2}$$

where:

- n = the total number of data
- s^2 = sample variance
- σ^2 = population variance

You may think of s as the random variable in this test. The number of degrees of freedom is $df = n - 1$. **A test of a single variance may be right-tailed, left-tailed, or two-tailed.** Example 11.10 will show you how to set up the null and alternative hypotheses. The null and alternative hypotheses contain statements about the population variance.

Example 11.10

Math instructors are not only interested in how their students do on exams, on average, but how the exam scores vary. To many instructors, the variance (or standard deviation) may be more important than the average.

Suppose a math instructor believes that the standard deviation for his final exam is five points. One of his best students thinks otherwise. The student claims that the standard deviation is more than five points. If the student were to conduct a hypothesis test, what would the null and alternative hypotheses be?

Solution 11.10

Even though we are given the population standard deviation, we can set up the test using the population variance as follows.

- H_0: $\sigma^2 = 5^2$
- H_a: $\sigma^2 > 5^2$

Try It Σ

11.10 A SCUBA instructor wants to record the collective depths each of his students dives during their checkout. He is interested in how the depths vary, even though everyone should have been at the same depth. He believes the standard deviation is three feet. His assistant thinks the standard deviation is less than three feet. If the instructor were to conduct a test, what would the null and alternative hypotheses be?

Example 11.11

With individual lines at its various windows, a post office finds that the standard deviation for normally distributed waiting times for customers on Friday afternoon is 7.2 minutes. The post office experiments with a single, main waiting line and finds that for a random sample of 25 customers, the waiting times for customers have a standard deviation of 3.5 minutes.

With a significance level of 5%, test the claim that **a single line causes lower variation among waiting times (shorter waiting times) for customers**.

Solution 11.11

Since the claim is that a single line causes less variation, this is a test of a single variance. The parameter is the population variance, σ^2, or the population standard deviation, σ.

Random Variable: The sample standard deviation, *s*, is the random variable. Let s = standard deviation for the waiting times.

- H_0: $\sigma^2 = 7.2^2$
- H_a: $\sigma^2 < 7.2^2$

The word **"less"** tells you this is a left-tailed test.

Distribution for the test: χ^2_{24}, where:

- n = the number of customers sampled
- $df = n – 1 = 25 – 1 = 24$

Calculate the test statistic:

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2} = \frac{(25 - 1)(3.5)^2}{7.2^2} = 5.67$$

where $n = 25$, $s = 3.5$, and $\sigma = 7.2$.

Graph:

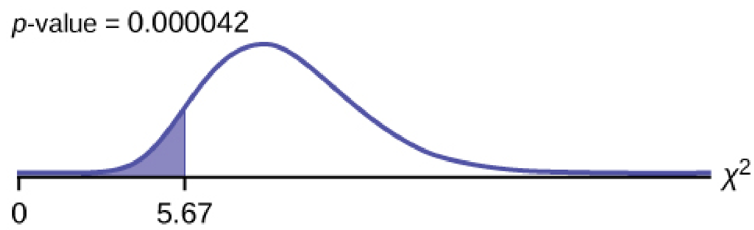

Figure 11.8

Probability statement: p-value = P (χ^2 < 5.67) = 0.000042

Compare α and the p-value:
α = 0.05; p-value = 0.000042; α > p-value

Make a decision: Since α > p-value, reject H_0. This means that you reject $\sigma^2 = 7.2^2$. In other words, you do not think the variation in waiting times is 7.2 minutes; you think the variation in waiting times is less.

Conclusion: At a 5% level of significance, from the data, there is sufficient evidence to conclude that a single line causes a lower variation among the waiting times **or** with a single line, the customer waiting times vary less than 7.2 minutes.

Using the TI-83, 83+, 84, 84+ Calculator

In `2nd DISTR`, use `7:χ2cdf`. The syntax is `(lower, upper, df)` for the parameter list. For **Example 11.11**, `χ2cdf(-1E99,5.67,24)`. The p-value = 0.000042.

Try It Σ

11.11 The FCC conducts broadband speed tests to measure how much data per second passes between a consumer's computer and the internet. As of August of 2012, the standard deviation of Internet speeds across Internet Service Providers (ISPs) was 12.2 percent. Suppose a sample of 15 ISPs is taken, and the standard deviation is 13.2. An analyst claims that the standard deviation of speeds is more than what was reported. State the null and alternative hypotheses, compute the degrees of freedom, the test statistic, sketch the graph of the p-value, and draw a conclusion. Test at the 1% significance level.

11.7 | Lab 1: Chi-Square Goodness-of-Fit

Stats Lab

11.1 Lab 1: Chi-Square Goodness-of-Fit

Class Time:

Names:

Student Learning Outcome

- The student will evaluate data collected to determine if they fit either the uniform or exponential distributions.

Collect the Data

Go to your local supermarket. Ask 30 people as they leave for the total amount on their grocery receipts. (Or, ask three cashiers for the last ten amounts. Be sure to include the express lane, if it is open.)

NOTE

You may need to combine two categories so that each cell has an expected value of at least five.

1. Record the values.

__________	__________	__________	__________	__________
__________	__________	__________	__________	__________
__________	__________	__________	__________	__________
__________	__________	__________	__________	__________
__________	__________	__________	__________	__________
__________	__________	__________	__________	__________

Table 11.23

2. Construct a histogram of the data. Make five to six intervals. Sketch the graph using a ruler and pencil. Scale the axes.

Figure 11.9

3. Calculate the following:
   a. $\bar{x}$ = ________
   b. s = ________

c. s^2 = ________

Uniform Distribution

Test to see if grocery receipts follow the uniform distribution.

1. Using your lowest and highest values, $X \sim U$ (_______, _______)
2. Divide the distribution into fifths.
3. Calculate the following:
   a. lowest value = _________
   b. 20th percentile = _________
   c. 40th percentile = _________
   d. 60th percentile = _________
   e. 80th percentile = _________
   f. highest value = _________
4. For each fifth, count the observed number of receipts and record it. Then determine the expected number of receipts and record that.

Fifth	Observed	Expected
1st		
2nd		
3rd		
4th		
5th		

Table 11.24

5. H_0: ________
6. H_a: ________
7. What distribution should you use for a hypothesis test?
8. Why did you choose this distribution?
9. Calculate the test statistic.
10. Find the *p*-value.
11. Sketch a graph of the situation. Label and scale the *x*-axis. Shade the area corresponding to the *p*-value.

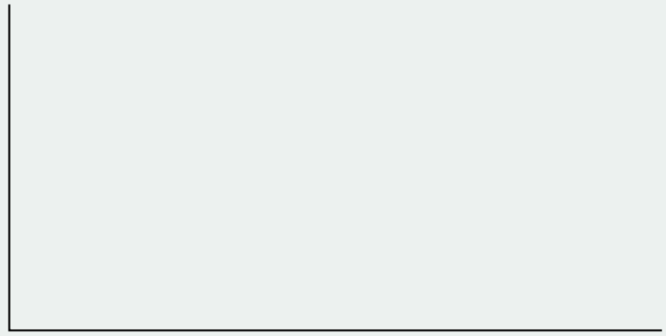

Figure 11.10

12. State your decision.
13. State your conclusion in a complete sentence.

Exponential Distribution

Test to see if grocery receipts follow the exponential distribution with decay parameter $\frac{1}{\bar{x}}$.

1. Using $\frac{1}{\bar{x}}$ as the decay parameter, $X \sim Exp$(__________).
2. Calculate the following:
   a. lowest value = ________
   b. first quartile = ________
   c. 37^{th} percentile = ________
   d. median = ________
   e. 63^{rd} percentile = ________
   f. 3^{rd} quartile = ________
   g. highest value = ________
3. For each cell, count the observed number of receipts and record it. Then determine the expected number of receipts and record that.

Cell	Observed	Expected
1^{st}		
2^{nd}		
3^{rd}		
4^{th}		
5^{th}		
6^{th}		

Table 11.25

4. H_0: ________
5. H_a: ________

6. What distribution should you use for a hypothesis test?
7. Why did you choose this distribution?
8. Calculate the test statistic.
9. Find the *p*-value.
10. Sketch a graph of the situation. Label and scale the *x*-axis. Shade the area corresponding to the *p*-value.

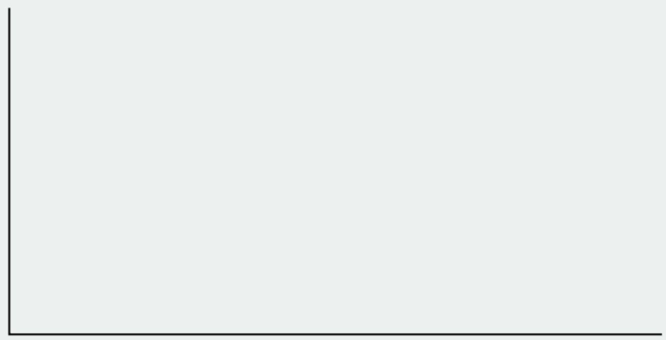

Figure 11.11

11. State your decision.
12. State your conclusion in a complete sentence.

Discussion Questions

1. Did your data fit either distribution? If so, which?
2. In general, do you think it's likely that data could fit more than one distribution? In complete sentences, explain why or why not.

11.8 | Lab 2: Chi-Square Test of Independence

Stats Lab

11.2 Lab 2: Chi-Square Test of Independence

Class Time:

Names:

Student Learning Outcome

- The student will evaluate if there is a significant relationship between favorite type of snack and gender.

Collect the Data

1. Using your class as a sample, complete the following chart. Ask each other what your favorite snack is, then total the results.

NOTE

You may need to combine two food categories so that each cell has an expected value of at least five.

	sweets (candy & baked goods)	ice cream	chips & pretzels	fruits & vegetables	Total
male					
female					
Total					

Table 11.26 Favorite type of snack

2. Looking at **Table 11.26**, does it appear to you that there is a dependence between gender and favorite type of snack food? Why or why not?

Hypothesis Test

Conduct a hypothesis test to determine if the factors are independent:

1. H_0: ________
2. H_a: ________
3. What distribution should you use for a hypothesis test?
4. Why did you choose this distribution?
5. Calculate the test statistic.
6. Find the *p*-value.
7. Sketch a graph of the situation. Label and scale the *x*-axis. Shade the area corresponding to the *p*-value.

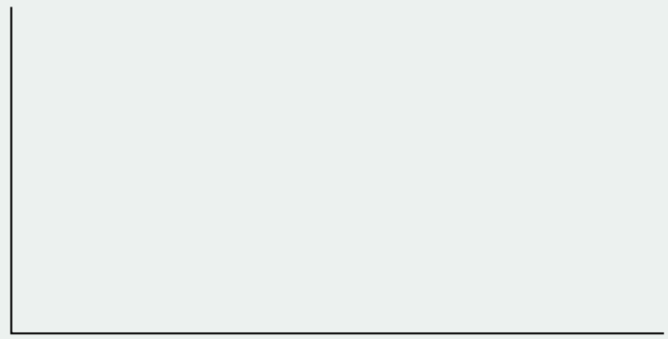

Figure 11.12

8. State your decision.
9. State your conclusion in a complete sentence.

Discussion Questions

1. Is the conclusion of your study the same as or different from your answer to answer to question two under **Collect the Data**?
2. Why do you think that occurred?

KEY TERMS

Contingency Table a table that displays sample values for two different factors that may be dependent or contingent on one another; it facilitates determining conditional probabilities.

CHAPTER REVIEW

11.1 Facts About the Chi-Square Distribution

The chi-square distribution is a useful tool for assessment in a series of problem categories. These problem categories include primarily (i) whether a data set fits a particular distribution, (ii) whether the distributions of two populations are the same, (iii) whether two events might be independent, and (iv) whether there is a different variability than expected within a population.

An important parameter in a chi-square distribution is the degrees of freedom *df* in a given problem. The random variable in the chi-square distribution is the sum of squares of *df* standard normal variables, which must be independent. The key characteristics of the chi-square distribution also depend directly on the degrees of freedom.

The chi-square distribution curve is skewed to the right, and its shape depends on the degrees of freedom *df*. For $df > 90$, the curve approximates the normal distribution. Test statistics based on the chi-square distribution are always greater than or equal to zero. Such application tests are almost always right-tailed tests.

11.2 Goodness-of-Fit Test

To assess whether a data set fits a specific distribution, you can apply the goodness-of-fit hypothesis test that uses the chi-square distribution. The null hypothesis for this test states that the data come from the assumed distribution. The test compares observed values against the values you would expect to have if your data followed the assumed distribution. The test is almost always right-tailed. Each observation or cell category must have an expected value of at least five.

11.3 Test of Independence

To assess whether two factors are independent or not, you can apply the test of independence that uses the chi-square distribution. The null hypothesis for this test states that the two factors are independent. The test compares observed values to expected values. The test is right-tailed. Each observation or cell category must have an expected value of at least 5.

11.4 Test for Homogeneity

To assess whether two data sets are derived from the same distribution—which need not be known, you can apply the test for homogeneity that uses the chi-square distribution. The null hypothesis for this test states that the populations of the two data sets come from the same distribution. The test compares the observed values against the expected values if the two populations followed the same distribution. The test is right-tailed. Each observation or cell category must have an expected value of at least five.

11.5 Comparison of the Chi-Square Tests

The goodness-of-fit test is typically used to determine if data fits a particular distribution. The test of independence makes use of a contingency table to determine the independence of two factors. The test for homogeneity determines whether two populations come from the same distribution, even if this distribution is unknown.

11.6 Test of a Single Variance

To test variability, use the chi-square test of a single variance. The test may be left-, right-, or two-tailed, and its hypotheses are always expressed in terms of the variance (or standard deviation).

FORMULA REVIEW

11.1 Facts About the Chi-Square Distribution

$\chi^2 = (Z_1)^2 + (Z_2)^2 + \ldots (Z_{df})^2$ chi-square distribution random variable

$\mu_{\chi^2} = df$ chi-square distribution population mean

$\sigma_{\chi^2} = \sqrt{2(df)}$ Chi-Square distribution population standard deviation

11.2 Goodness-of-Fit Test

$\sum_k \frac{(O-E)^2}{E}$ goodness-of-fit test statistic where:

O: observed values
E: expected values

k: number of different data cells or categories

$df = k - 1$ degrees of freedom

11.3 Test of Independence

Test of Independence

- The number of degrees of freedom is equal to (number of columns - 1)(number of rows - 1).
- The test statistic is $\sum_{(i \cdot j)} \frac{(O-E)^2}{E}$ where *O* = observed values, *E* = expected values, *i* = the number of rows in the table, and *j* = the number of columns in the table.
- If the null hypothesis is true, the expected number $E = \frac{\text{(row total)(column total)}}{\text{total surveyed}}$.

11.4 Test for Homogeneity

$\sum_{i \cdot j} \frac{(O-E)^2}{E}$ Homogeneity test statistic where: *O* = observed values
E = expected values
i = number of rows in data contingency table
j = number of columns in data contingency table

$df = (i - 1)(j - 1)$ Degrees of freedom

11.6 Test of a Single Variance

$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$ Test of a single variance statistic where:
n: sample size
s: sample standard deviation
σ: population standard deviation

$df = n - 1$ Degrees of freedom

Test of a Single Variance

- Use the test to determine variation.
- The degrees of freedom is the number of samples – 1.
- The test statistic is $\frac{(n-1) \cdot s^2}{\sigma^2}$, where *n* = the total number of data, s^2 = sample variance, and σ^2 = population variance.
- The test may be left-, right-, or two-tailed.

PRACTICE

11.1 Facts About the Chi-Square Distribution

1. If the number of degrees of freedom for a chi-square distribution is 25, what is the population mean and standard deviation?

2. If $df > 90$, the distribution is _____________. If $df = 15$, the distribution is ________________.

3. When does the chi-square curve approximate a normal distribution?

4. Where is μ located on a chi-square curve?

5. Is it more likely the *df* is 90, 20, or two in the graph?

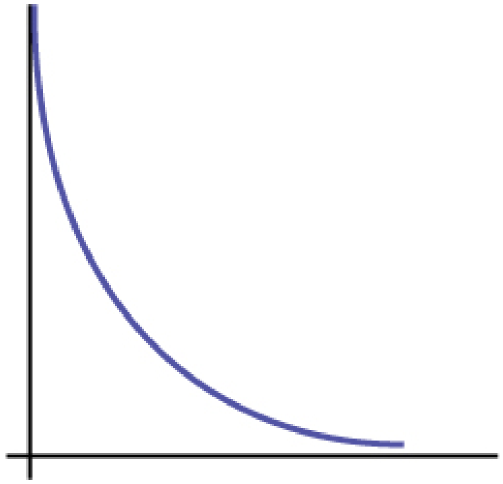

Figure 11.13

11.2 Goodness-of-Fit Test

Determine the appropriate test to be used in the next three exercises.

6. An archeologist is calculating the distribution of the frequency of the number of artifacts she finds in a dig site. Based on previous digs, the archeologist creates an expected distribution broken down by grid sections in the dig site. Once the site has been fully excavated, she compares the actual number of artifacts found in each grid section to see if her expectation was accurate.

7. An economist is deriving a model to predict outcomes on the stock market. He creates a list of expected points on the stock market index for the next two weeks. At the close of each day's trading, he records the actual points on the index. He wants to see how well his model matched what actually happened.

8. A personal trainer is putting together a weight-lifting program for her clients. For a 90-day program, she expects each client to lift a specific maximum weight each week. As she goes along, she records the actual maximum weights her clients lifted. She wants to know how well her expectations met with what was observed.

Use the following information to answer the next five exercises: A teacher predicts that the distribution of grades on the final exam will be and they are recorded in **Table 11.27**.

Grade	Proportion
A	0.25
B	0.30
C	0.35
D	0.10

Table 11.27

The actual distribution for a class of 20 is in **Table 11.28**.

Grade	Frequency
A	7
B	7
C	5
D	1

Table 11.28

9. $df =$ ______

10. State the null and alternative hypotheses.

11. χ^2 test statistic = ______

12. p-value = ______

13. At the 5% significance level, what can you conclude?

Use the following information to answer the next nine exercises: The following data are real. The cumulative number of AIDS cases reported for Santa Clara County is broken down by ethnicity as in **Table 11.29**.

Ethnicity	Number of Cases
White	2,229
Hispanic	1,157
Black/African-American	457
Asian, Pacific Islander	232
	Total = 4,075

Table 11.29

The percentage of each ethnic group in Santa Clara County is as in **Table 11.30**.

Ethnicity	Percentage of total county population	Number expected (round to two decimal places)
White	42.9%	1748.18
Hispanic	26.7%	
Black/African-American	2.6%	
Asian, Pacific Islander	27.8%	
	Total = 100%	

Table 11.30

14. If the ethnicities of AIDS victims followed the ethnicities of the total county population, fill in the expected number of cases per ethnic group.

Perform a goodness-of-fit test to determine whether the occurrence of AIDS cases follows the ethnicities of the general population of Santa Clara County.

15. H_0: _______

16. H_a: _______

17. Is this a right-tailed, left-tailed, or two-tailed test?

18. degrees of freedom = _______

19. χ^2 test statistic = _______

20. p-value = _______

21. Graph the situation. Label and scale the horizontal axis. Mark the mean and test statistic. Shade in the region corresponding to the p-value.

Figure 11.14

Let $\alpha = 0.05$

Decision: ________________

Reason for the Decision: ________________

Conclusion (write out in complete sentences): ________________

22. Does it appear that the pattern of AIDS cases in Santa Clara County corresponds to the distribution of ethnic groups in this county? Why or why not?

11.3 Test of Independence

Determine the appropriate test to be used in the next three exercises.

23. A pharmaceutical company is interested in the relationship between age and presentation of symptoms for a common viral infection. A random sample is taken of 500 people with the infection across different age groups.

24. The owner of a baseball team is interested in the relationship between player salaries and team winning percentage. He takes a random sample of 100 players from different organizations.

25. A marathon runner is interested in the relationship between the brand of shoes runners wear and their run times. She takes a random sample of 50 runners and records their run times as well as the brand of shoes they were wearing.

Use the following information to answer the next seven exercises: Transit Railroads is interested in the relationship between travel distance and the ticket class purchased. A random sample of 200 passengers is taken. **Table 11.31** shows the results. The railroad wants to know if a passenger's choice in ticket class is independent of the distance they must travel.

Traveling Distance	Third class	Second class	First class	Total
1–100 miles	21	14	6	41
101–200 miles	18	16	8	42
201–300 miles	16	17	15	48
301–400 miles	12	14	21	47
401–500 miles	6	6	10	22
Total	73	67	60	200

Table 11.31

26. State the hypotheses.
H_0: _______
H_a: _______

27. df = _______

28. How many passengers are expected to travel between 201 and 300 miles and purchase second-class tickets?

29. How many passengers are expected to travel between 401 and 500 miles and purchase first-class tickets?

30. What is the test statistic?

31. What is the *p*-value?

32. What can you conclude at the 5% level of significance?

Use the following information to answer the next eight exercises: An article in the New England Journal of Medicine, discussed a study on smokers in California and Hawaii. In one part of the report, the self-reported ethnicity and smoking levels per day were given. Of the people smoking at most ten cigarettes per day, there were 9,886 African Americans, 2,745 Native Hawaiians, 12,831 Latinos, 8,378 Japanese Americans and 7,650 whites. Of the people smoking 11 to 20 cigarettes per day, there were 6,514 African Americans, 3,062 Native Hawaiians, 4,932 Latinos, 10,680 Japanese Americans, and 9,877 whites. Of the people smoking 21 to 30 cigarettes per day, there were 1,671 African Americans, 1,419 Native Hawaiians, 1,406 Latinos, 4,715 Japanese Americans, and 6,062 whites. Of the people smoking at least 31 cigarettes per day, there were 759 African Americans, 788 Native Hawaiians, 800 Latinos, 2,305 Japanese Americans, and 3,970 whites.

33. Complete the table.

Smoking Level Per Day	African American	Native Hawaiian	Latino	Japanese Americans	White	TOTALS
1-10						
11-20						
21-30						
31+						
TOTALS						

Table 11.32 Smoking Levels by Ethnicity (Observed)

34. State the hypotheses.
H_0: ________
H_a: ________

35. Enter expected values in **Table 11.32**. Round to two decimal places.

Calculate the following values:

36. df = ________

37. χ^2 test statistic = ______

38. *p*-value = ______

39. Is this a right-tailed, left-tailed, or two-tailed test? Explain why.

40. Graph the situation. Label and scale the horizontal axis. Mark the mean and test statistic. Shade in the region corresponding to the *p*-value.

Figure 11.15

State the decision and conclusion (in a complete sentence) for the following preconceived levels of α.

41. $\alpha = 0.05$

a. Decision: ____________________
b. Reason for the decision: ____________________
c. Conclusion (write out in a complete sentence): ____________________

42. $\alpha = 0.01$

a. Decision: ____________________
b. Reason for the decision: ____________________
c. Conclusion (write out in a complete sentence): ____________________

11.4 Test for Homogeneity

43. A math teacher wants to see if two of her classes have the same distribution of test scores. What test should she use?

44. What are the null and alternative hypotheses for **Exercise 11.43**?

45. A market researcher wants to see if two different stores have the same distribution of sales throughout the year. What type of test should he use?

46. A meteorologist wants to know if East and West Australia have the same distribution of storms. What type of test should she use?

47. What condition must be met to use the test for homogeneity?

Use the following information to answer the next five exercises: Do private practice doctors and hospital doctors have the same distribution of working hours? Suppose that a sample of 100 private practice doctors and 150 hospital doctors are selected at random and asked about the number of hours a week they work. The results are shown in **Table 11.33**.

	20–30	30–40	40–50	50–60
Private Practice	16	40	38	6
Hospital	8	44	59	39

Table 11.33

48. State the null and alternative hypotheses.

49. df = ________

50. What is the test statistic?

51. What is the p-value?

52. What can you conclude at the 5% significance level?

11.5 Comparison of the Chi-Square Tests

53. Which test do you use to decide whether an observed distribution is the same as an expected distribution?

54. What is the null hypothesis for the type of test from **Exercise 11.53**?

55. Which test would you use to decide whether two factors have a relationship?

56. Which test would you use to decide if two populations have the same distribution?

57. How are tests of independence similar to tests for homogeneity?

58. How are tests of independence different from tests for homogeneity?

11.6 Test of a Single Variance

Use the following information to answer the next three exercises: An archer's standard deviation for his hits is six (data is measured in distance from the center of the target). An observer claims the standard deviation is less.

59. What type of test should be used?

60. State the null and alternative hypotheses.

61. Is this a right-tailed, left-tailed, or two-tailed test?

Use the following information to answer the next three exercises: The standard deviation of heights for students in a school

is 0.81. A random sample of 50 students is taken, and the standard deviation of heights of the sample is 0.96. A researcher in charge of the study believes the standard deviation of heights for the school is greater than 0.81.

62. What type of test should be used?

63. State the null and alternative hypotheses.

64. *df* = ________

Use the following information to answer the next four exercises: The average waiting time in a doctor's office varies. The standard deviation of waiting times in a doctor's office is 3.4 minutes. A random sample of 30 patients in the doctor's office has a standard deviation of waiting times of 4.1 minutes. One doctor believes the variance of waiting times is greater than originally thought.

65. What type of test should be used?

66. What is the test statistic?

67. What is the *p*-value?

68. What can you conclude at the 5% significance level?

HOMEWORK

11.1 Facts About the Chi-Square Distribution

Decide whether the following statements are true or false.

69. As the number of degrees of freedom increases, the graph of the chi-square distribution looks more and more symmetrical.

70. The standard deviation of the chi-square distribution is twice the mean.

71. The mean and the median of the chi-square distribution are the same if *df* = 24.

11.2 Goodness-of-Fit Test

For each problem, use a solution sheet to solve the hypothesis test problem. Go to **Appendix E** *for the chi-square solution sheet. Round expected frequency to two decimal places.*

72. A six-sided die is rolled 120 times. Fill in the expected frequency column. Then, conduct a hypothesis test to determine if the die is fair. The data in **Table 11.34** are the result of the 120 rolls.

Face Value	Frequency	Expected Frequency
1	15	
2	29	
3	16	
4	15	
5	30	
6	15	

Table 11.34

73. The marital status distribution of the U.S. male population, ages 15 and older, is as shown in **Table 11.35**.

Marital Status	Percent	Expected Frequency
never married	31.3	
married	56.1	
widowed	2.5	

Table 11.35

Marital Status	Percent	Expected Frequency
divorced/separated	10.1	

Table 11.35

Suppose that a random sample of 400 U.S. young adult males, 18 to 24 years old, yielded the following frequency distribution. We are interested in whether this age group of males fits the distribution of the U.S. adult population. Calculate the frequency one would expect when surveying 400 people. Fill in Table 11.36, rounding to two decimal places.

Marital Status	Frequency
never married	140
married	238
widowed	2
divorced/separated	20

Table 11.36

Use the following information to answer the next two exercises: The columns in Table 11.37 contain the Race/Ethnicity of U.S. Public Schools for a recent year, the percentages for the Advanced Placement Examinee Population for that class, and the Overall Student Population. Suppose the right column contains the result of a survey of 1,000 local students from that year who took an AP Exam.

Race/Ethnicity	AP Examinee Population	Overall Student Population	Survey Frequency
Asian, Asian American, or Pacific Islander	10.2%	5.4%	113
Black or African-American	8.2%	14.5%	94
Hispanic or Latino	15.5%	15.9%	136
American Indian or Alaska Native	0.6%	1.2%	10
White	59.4%	61.6%	604
Not reported/other	6.1%	1.4%	43

Table 11.37

74. Perform a goodness-of-fit test to determine whether the local results follow the distribution of the U.S. overall student population based on ethnicity.

75. Perform a goodness-of-fit test to determine whether the local results follow the distribution of U.S. AP examinee population, based on ethnicity.

76. The City of South Lake Tahoe, CA, has an Asian population of 1,419 people, out of a total population of 23,609. Suppose that a survey of 1,419 self-reported Asians in the Manhattan, NY, area yielded the data in Table 11.38. Conduct a goodness-of-fit test to determine if the self-reported sub-groups of Asians in the Manhattan area fit that of the Lake Tahoe area.

Race	Lake Tahoe Frequency	Manhattan Frequency
Asian Indian	131	174
Chinese	118	557
Filipino	1,045	518

Table 11.38

Race	Lake Tahoe Frequency	Manhattan Frequency
Japanese	80	54
Korean	12	29
Vietnamese	9	21
Other	24	66

Table 11.38

Use the following information to answer the next two exercises: UCLA conducted a survey of more than 263,000 college freshmen from 385 colleges in fall 2005. The results of students' expected majors by gender were reported in *The Chronicle of Higher Education (2/2/2006)*. Suppose a survey of 5,000 graduating females and 5,000 graduating males was done as a follow-up last year to determine what their actual majors were. The results are shown in the tables for **Exercise 11.77** and **Exercise 11.78**. The second column in each table does not add to 100% because of rounding.

77. Conduct a goodness-of-fit test to determine if the actual college majors of graduating females fit the distribution of their expected majors.

Major	Women - Expected Major	Women - Actual Major
Arts & Humanities	14.0%	670
Biological Sciences	8.4%	410
Business	13.1%	685
Education	13.0%	650
Engineering	2.6%	145
Physical Sciences	2.6%	125
Professional	18.9%	975
Social Sciences	13.0%	605
Technical	0.4%	15
Other	5.8%	300
Undecided	8.0%	420

Table 11.39

78. Conduct a goodness-of-fit test to determine if the actual college majors of graduating males fit the distribution of their expected majors.

Major	Men - Expected Major	Men - Actual Major
Arts & Humanities	11.0%	600
Biological Sciences	6.7%	330
Business	22.7%	1130
Education	5.8%	305
Engineering	15.6%	800
Physical Sciences	3.6%	175
Professional	9.3%	460
Social Sciences	7.6%	370
Technical	1.8%	90

Table 11.40

Major	Men - Expected Major	Men - Actual Major
Other	8.2%	400
Undecided	6.6%	340

Table 11.40

Read the statement and decide whether it is true or false.

79. In a goodness-of-fit test, the expected values are the values we would expect if the null hypothesis were true.

80. In general, if the observed values and expected values of a goodness-of-fit test are not close together, then the test statistic can get very large and on a graph will be way out in the right tail.

81. Use a goodness-of-fit test to determine if high school principals believe that students are absent equally during the week or not.

82. The test to use to determine if a six-sided die is fair is a goodness-of-fit test.

83. In a goodness-of fit test, if the *p*-value is 0.0113, in general, do not reject the null hypothesis.

84. A sample of 212 commercial businesses was surveyed for recycling one commodity; a commodity here means any one type of recyclable material such as plastic or aluminum. **Table 11.41** shows the business categories in the survey, the sample size of each category, and the number of businesses in each category that recycle one commodity. Based on the study, on average half of the businesses were expected to be recycling one commodity. As a result, the last column shows the expected number of businesses in each category that recycle one commodity. At the 5% significance level, perform a hypothesis test to determine if the observed number of businesses that recycle one commodity follows the uniform distribution of the expected values.

Business Type	Number in class	Observed Number that recycle one commodity	Expected number that recycle one commodity
Office	35	19	17.5
Retail/ Wholesale	48	27	24
Food/ Restaurants	53	35	26.5
Manufacturing/ Medical	52	21	26
Hotel/Mixed	24	9	12

Table 11.41

85. **Table 11.42** contains information from a survey among 499 participants classified according to their age groups. The second column shows the percentage of obese people per age class among the study participants. The last column comes from a different study at the national level that shows the corresponding percentages of obese people in the same age classes in the USA. Perform a hypothesis test at the 5% significance level to determine whether the survey participants are a representative sample of the USA obese population.

Age Class (Years)	Obese (Percentage)	Expected USA average (Percentage)
20–30	75.0	32.6
31–40	26.5	32.6
41–50	13.6	36.6
51–60	21.9	36.6
61–70	21.0	39.7

Table 11.42

11.3 Test of Independence

For each problem, use a solution sheet to solve the hypothesis test problem. Go to **Appendix E** *for the chi-square solution sheet. Round expected frequency to two decimal places.*

86. A recent debate about where in the United States skiers believe the skiing is best prompted the following survey. Test to see if the best ski area is independent of the level of the skier.

U.S. Ski Area	Beginner	Intermediate	Advanced
Tahoe	20	30	40
Utah	10	30	60
Colorado	10	40	50

Table 11.43

87. Car manufacturers are interested in whether there is a relationship between the size of car an individual drives and the number of people in the driver's family (that is, whether car size and family size are independent). To test this, suppose that 800 car owners were randomly surveyed with the results in **Table 11.44**. Conduct a test of independence.

Family Size	Sub & Compact	Mid-size	Full-size	Van & Truck
1	20	35	40	35
2	20	50	70	80
3–4	20	50	100	90
5+	20	30	70	70

Table 11.44

88. College students may be interested in whether or not their majors have any effect on starting salaries after graduation. Suppose that 300 recent graduates were surveyed as to their majors in college and their starting salaries after graduation. **Table 11.45** shows the data. Conduct a test of independence.

Major	< \$50,000	\$50,000 – \$68,999	\$69,000 +
English	5	20	5
Engineering	10	30	60
Nursing	10	15	15
Business	10	20	30
Psychology	20	30	20

Table 11.45

89. Some travel agents claim that honeymoon hot spots vary according to age of the bride. Suppose that 280 recent brides were interviewed as to where they spent their honeymoons. The information is given in **Table 11.46**. Conduct a test of independence.

Location	20–29	30–39	40–49	50 and over
Niagara Falls	15	25	25	20
Poconos	15	25	25	10
Europe	10	25	15	5

Table 11.46

Location	20–29	30–39	40–49	50 and over
Virgin Islands	20	25	15	5

Table 11.46

90. A manager of a sports club keeps information concerning the main sport in which members participate and their ages. To test whether there is a relationship between the age of a member and his or her choice of sport, 643 members of the sports club are randomly selected. Conduct a test of independence.

Sport	18 - 25	26 - 30	31 - 40	41 and over
racquetball	42	58	30	46
tennis	58	76	38	65
swimming	72	60	65	33

Table 11.47

91. A major food manufacturer is concerned that the sales for its skinny french fries have been decreasing. As a part of a feasibility study, the company conducts research into the types of fries sold across the country to determine if the type of fries sold is independent of the area of the country. The results of the study are shown in **Table 11.48**. Conduct a test of independence.

Type of Fries	Northeast	South	Central	West
skinny fries	70	50	20	25
curly fries	100	60	15	30
steak fries	20	40	10	10

Table 11.48

92. According to Dan Lenard, an independent insurance agent in the Buffalo, N.Y. area, the following is a breakdown of the amount of life insurance purchased by males in the following age groups. He is interested in whether the age of the male and the amount of life insurance purchased are independent events. Conduct a test for independence.

Age of Males	None	< $200,000	$200,000–$400,000	$401,001–$1,000,000	$1,000,001+
20–29	40	15	40	0	5
30–39	35	5	20	20	10
40–49	20	0	30	0	30
50+	40	30	15	15	10

Table 11.49

93. Suppose that 600 thirty-year-olds were surveyed to determine whether or not there is a relationship between the level of education an individual has and salary. Conduct a test of independence.

Annual Salary	Not a high school graduate	High school graduate	College graduate	Masters or doctorate
< $30,000	15	25	10	5
$30,000–$40,000	20	40	70	30

Table 11.50

Annual Salary	Not a high school graduate	High school graduate	College graduate	Masters or doctorate
\$40,000–\$50,000	10	20	40	55
\$50,000–\$60,000	5	10	20	60
\$60,000+	0	5	10	150

Table 11.50

Read the statement and decide whether it is true or false.

94. The number of degrees of freedom for a test of independence is equal to the sample size minus one.

95. The test for independence uses tables of observed and expected data values.

96. The test to use when determining if the college or university a student chooses to attend is related to his or her socioeconomic status is a test for independence.

97. In a test of independence, the expected number is equal to the row total multiplied by the column total divided by the total surveyed.

98. An ice cream maker performs a nationwide survey about favorite flavors of ice cream in different geographic areas of the U.S. Based on **Table 11.51**, do the numbers suggest that geographic location is independent of favorite ice cream flavors? Test at the 5% significance level.

U.S. region/ Flavor	Strawberry	Chocolate	Vanilla	Rocky Road	Mint Chocolate Chip	Pistachio	Row total
West	12	21	22	19	15	8	97
Midwest	10	32	22	11	15	6	96
East	8	31	27	8	15	7	96
South	15	28	30	8	15	6	102
Column Total	45	112	101	46	60	27	391

Table 11.51

99. **Table 11.52** provides a recent survey of the youngest online entrepreneurs whose net worth is estimated at one million dollars or more. Their ages range from 17 to 30. Each cell in the table illustrates the number of entrepreneurs who correspond to the specific age group and their net worth. Are the ages and net worth independent? Perform a test of independence at the 5% significance level.

Age Group\ Net Worth Value (in millions of US dollars)	1–5	6–24	≥25	Row Total
17–25	8	7	5	20
26–30	6	5	9	20
Column Total	14	12	14	40

Table 11.52

100. A 2013 poll in California surveyed people about taxing sugar-sweetened beverages. The results are presented in **Table 11.53**, and are classified by ethnic group and response type. Are the poll responses independent of the participants' ethnic group? Conduct a test of independence at the 5% significance level.

Opinion/ Ethnicity	Asian-American	White/Non-Hispanic	African-American	Latino	Row Total
Against tax	48	433	41	160	628
In Favor of tax	54	234	24	147	459
No opinion	16	43	16	19	84
Column Total	118	710	71	272	1171

Table 11.53

11.4 Test for Homogeneity

For each word problem, use a solution sheet to solve the hypothesis test problem. Go to **Appendix E** *for the chi-square solution sheet. Round expected frequency to two decimal places.*

101. A psychologist is interested in testing whether there is a difference in the distribution of personality types for business majors and social science majors. The results of the study are shown in **Table 11.54**. Conduct a test of homogeneity. Test at a 5% level of significance.

	Open	Conscientious	Extrovert	Agreeable	Neurotic
Business	41	52	46	61	58
Social Science	72	75	63	80	65

Table 11.54

102. Do men and women select different breakfasts? The breakfasts ordered by randomly selected men and women at a popular breakfast place is shown in **Table 11.55**. Conduct a test for homogeneity at a 5% level of significance.

	French Toast	Pancakes	Waffles	Omelettes
Men	47	35	28	53
Women	65	59	55	60

Table 11.55

103. A fisherman is interested in whether the distribution of fish caught in Green Valley Lake is the same as the distribution of fish caught in Echo Lake. Of the 191 randomly selected fish caught in Green Valley Lake, 105 were rainbow trout, 27 were other trout, 35 were bass, and 24 were catfish. Of the 293 randomly selected fish caught in Echo Lake, 115 were rainbow trout, 58 were other trout, 67 were bass, and 53 were catfish. Perform a test for homogeneity at a 5% level of significance.

104. In 2007, the United States had 1.5 million homeschooled students, according to the U.S. National Center for Education Statistics. In **Table 11.56** you can see that parents decide to homeschool their children for different reasons, and some reasons are ranked by parents as more important than others. According to the survey results shown in the table, is the distribution of applicable reasons the same as the distribution of the most important reason? Provide your assessment at the 5% significance level. Did you expect the result you obtained?

Reasons for Homeschooling	Applicable Reason (in thousands of respondents)	Most Important Reason (in thousands of respondents)	Row Total
Concern about the environment of other schools	1,321	309	1,630

Table 11.56

Reasons for Homeschooling	Applicable Reason (in thousands of respondents)	Most Important Reason (in thousands of respondents)	Row Total
Dissatisfaction with academic instruction at other schools	1,096	258	1,354
To provide religious or moral instruction	1,257	540	1,797
Child has special needs, other than physical or mental	315	55	370
Nontraditional approach to child's education	984	99	1,083
Other reasons (e.g., finances, travel, family time, etc.)	485	216	701
Column Total	5,458	1,477	6,935

Table 11.56

105. When looking at energy consumption, we are often interested in detecting trends over time and how they correlate among different countries. The information in **Table 11.57** shows the average energy use (in units of kg of oil equivalent per capita) in the USA and the joint European Union countries (EU) for the six-year period 2005 to 2010. Do the energy use values in these two areas come from the same distribution? Perform the analysis at the 5% significance level.

Year	European Union	United States	Row Total
2010	3,413	7,164	10,557
2009	3,302	7,057	10,359
2008	3,505	7,488	10,993
2007	3,537	7,758	11,295
2006	3,595	7,697	11,292
2005	3,613	7,847	11,460
Column Total	45,011	20,965	65,976

Table 11.57

106. The Insurance Institute for Highway Safety collects safety information about all types of cars every year, and publishes a report of Top Safety Picks among all cars, makes, and models. **Table 11.58** presents the number of Top Safety Picks in six car categories for the two years 2009 and 2013. Analyze the table data to conclude whether the distribution of cars that earned the Top Safety Picks safety award has remained the same between 2009 and 2013. Derive your results at the 5% significance level.

Year \ Car Type	Small	Mid-Size	Large	Small SUV	Mid-Size SUV	Large SUV	Row Total
2009	12	22	10	10	27	6	87
2013	31	30	19	11	29	4	124
Column Total	43	52	29	21	56	10	211

Table 11.58

11.5 Comparison of the Chi-Square Tests

For each word problem, use a solution sheet to solve the hypothesis test problem. Go to **Appendix E** *for the chi-square solution sheet. Round expected frequency to two decimal places.*

107. Is there a difference between the distribution of community college statistics students and the distribution of university statistics students in what technology they use on their homework? Of some randomly selected community college students, 43 used a computer, 102 used a calculator with built in statistics functions, and 65 used a table from the textbook. Of some randomly selected university students, 28 used a computer, 33 used a calculator with built in statistics functions, and 40 used a table from the textbook. Conduct an appropriate hypothesis test using a 0.05 level of significance.

Read the statement and decide whether it is true or false.

108. If $df = 2$, the chi-square distribution has a shape that reminds us of the exponential.

11.6 Test of a Single Variance

Use the following information to answer the next twelve exercises: Suppose an airline claims that its flights are consistently on time with an average delay of at most 15 minutes. It claims that the average delay is so consistent that the variance is no more than 150 minutes. Doubting the consistency part of the claim, a disgruntled traveler calculates the delays for his next 25 flights. The average delay for those 25 flights is 22 minutes with a standard deviation of 15 minutes.

109. Is the traveler disputing the claim about the average or about the variance?

110. A sample standard deviation of 15 minutes is the same as a sample variance of __________ minutes.

111. Is this a right-tailed, left-tailed, or two-tailed test?

112. H_0: __________

113. df = ________

114. chi-square test statistic = ________

115. p-value = ________

116. Graph the situation. Label and scale the horizontal axis. Mark the mean and test statistic. Shade the p-value.

117. Let $\alpha = 0.05$
Decision: ________
Conclusion (write out in a complete sentence.): ________

118. How did you know to test the variance instead of the mean?

119. If an additional test were done on the claim of the average delay, which distribution would you use?

120. If an additional test were done on the claim of the average delay, but 45 flights were surveyed, which distribution would you use?

For each word problem, use a solution sheet to solve the hypothesis test problem. Go to **Appendix E** *for the chi-square solution sheet. Round expected frequency to two decimal places.*

121. A plant manager is concerned her equipment may need recalibrating. It seems that the actual weight of the 15 oz. cereal boxes it fills has been fluctuating. The standard deviation should be at most 0.5 oz. In order to determine if the machine needs to be recalibrated, 84 randomly selected boxes of cereal from the next day's production were weighed. The standard deviation of the 84 boxes was 0.54. Does the machine need to be recalibrated?

122. Consumers may be interested in whether the cost of a particular calculator varies from store to store. Based on surveying 43 stores, which yielded a sample mean of \$84 and a sample standard deviation of \$12, test the claim that the standard deviation is greater than \$15.

123. Isabella, an accomplished **Bay to Breakers** runner, claims that the standard deviation for her time to run the 7.5 mile race is at most three minutes. To test her claim, Rupinder looks up five of her race times. They are 55 minutes, 61 minutes, 58 minutes, 63 minutes, and 57 minutes.

124. Airline companies are interested in the consistency of the number of babies on each flight, so that they have adequate safety equipment. They are also interested in the variation of the number of babies. Suppose that an airline executive believes the average number of babies on flights is six with a variance of nine at most. The airline conducts a survey. The results of the 18 flights surveyed give a sample average of 6.4 with a sample standard deviation of 3.9. Conduct a hypothesis test of the airline executive's belief.

125. The number of births per woman in China is 1.6 down from 5.91 in 1966. This fertility rate has been attributed to the law passed in 1979 restricting births to one per woman. Suppose that a group of students studied whether or not the standard deviation of births per woman was greater than 0.75. They asked 50 women across China the number of births they had had. The results are shown in **Table 11.59**. Does the students' survey indicate that the standard deviation is greater than 0.75?

# of births	Frequency
0	5
1	30
2	10
3	5

Table 11.59

126. According to an avid aquarist, the average number of fish in a 20-gallon tank is 10, with a standard deviation of two. His friend, also an aquarist, does not believe that the standard deviation is two. She counts the number of fish in 15 other 20-gallon tanks. Based on the results that follow, do you think that the standard deviation is different from two? Data: 11; 10; 9; 10; 10; 11; 11; 10; 12; 9; 7; 9; 11; 10; 11

127. The manager of "Frenchies" is concerned that patrons are not consistently receiving the same amount of French fries with each order. The chef claims that the standard deviation for a ten-ounce order of fries is at most 1.5 oz., but the manager thinks that it may be higher. He randomly weighs 49 orders of fries, which yields a mean of 11 oz. and a standard deviation of two oz.

128. You want to buy a specific computer. A sales representative of the manufacturer claims that retail stores sell this computer at an average price of $1,249 with a very narrow standard deviation of $25. You find a website that has a price comparison for the same computer at a series of stores as follows: $1,299; $1,229.99; $1,193.08; $1,279; $1,224.95; $1,229.99; $1,269.95; $1,249. Can you argue that pricing has a larger standard deviation than claimed by the manufacturer? Use the 5% significance level. As a potential buyer, what would be the practical conclusion from your analysis?

129. A company packages apples by weight. One of the weight grades is Class A apples. Class A apples have a mean weight of 150 g, and there is a maximum allowed weight tolerance of 5% above or below the mean for apples in the same consumer package. A batch of apples is selected to be included in a Class A apple package. Given the following apple weights of the batch, does the fruit comply with the Class A grade weight tolerance requirements. Conduct an appropriate hypothesis test.

(a) at the 5% significance level

(b) at the 1% significance level

Weights in selected apple batch (in grams): 158; 167; 149; 169; 164; 139; 154; 150; 157; 171; 152; 161; 141; 166; 172;

BRINGING IT TOGETHER: HOMEWORK

130.

a. Explain why a goodness-of-fit test and a test of independence are generally right-tailed tests.
b. If you did a left-tailed test, what would you be testing?

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SOLUTIONS

1 mean = 25 and standard deviation = 7.0711

3 when the number of degrees of freedom is greater than 90

5 $df = 2$

7 a goodness-of-fit test

9 3

11 2.04

13 We decline to reject the null hypothesis. There is not enough evidence to suggest that the observed test scores are significantly different from the expected test scores.

15 H_0: the distribution of AIDS cases follows the ethnicities of the general population of Santa Clara County.

17 right-tailed

19 88,621

21 Graph: Check student’s solution. Decision: Reject the null hypothesis. Reason for the Decision: p-value < alpha Conclusion (write out in complete sentences): The make-up of AIDS cases does not fit the ethnicities of the general population of Santa Clara County.

23 a test of independence

25 a test of independence

27 8

29 6.6

31 0.0435

33

Smoking Level Per Day	African American	Native Hawaiian	Latino	Japanese Americans	White	Totals
1-10	9,886	2,745	12,831	8,378	7,650	41,490
11-20	6,514	3,062	4,932	10,680	9,877	35,065
21-30	1,671	1,419	1,406	4,715	6,062	15,273
31+	759	788	800	2,305	3,970	8,622
Totals	18,830	8,014	19,969	26,078	27,559	10,0450

Table 11.60

35

Smoking Level Per Day	African American	Native Hawaiian	Latino	Japanese Americans	White
1-10	7777.57	3310.11	8248.02	10771.29	11383.01
11-20	6573.16	2797.52	6970.76	9103.29	9620.27
21-30	2863.02	1218.49	3036.20	3965.05	4190.23
31+	1616.25	687.87	1714.01	2238.37	2365.49

Table 11.61

37 10,301.8

39 right

41

a. Reject the null hypothesis.
b. *p*-value < alpha
c. There is sufficient evidence to conclude that smoking level is dependent on ethnic group.

43 test for homogeneity

45 test for homogeneity

47 All values in the table must be greater than or equal to five.

49 3

51 0.00005

53 a goodness-of-fit test

55 a test for independence

57 Answers will vary. Sample answer: Tests of independence and tests for homogeneity both calculate the test statistic the same way $\sum_{(ij)} \frac{(O - E)^2}{E}$. In addition, all values must be greater than or equal to five.

59 a test of a single variance

61 a left-tailed test

63 H_0: $\sigma^2 = 0.81^2$; H_a: $\sigma^2 > 0.81^2$

65 a test of a single variance

67 0.0542

69 true

71 false

73

Marital Status	Percent	Expected Frequency
never married	31.3	125.2
married	56.1	224.4
widowed	2.5	10
divorced/separated	10.1	40.4

Table 11.62

a. The data fits the distribution.
b. The data does not fit the distribution.
c. 3
d. chi-square distribution with $df = 3$
e. 19.27
f. 0.0002
g. Check student's solution.
h. i. Alpha = 0.05
   ii. Decision: Reject null
   iii. Reason for decision: p-value < alpha
   iv. Conclusion: Data does not fit the distribution.

75
a. H_0: The local results follow the distribution of the U.S. AP examinee population
b. H_a: The local results do not follow the distribution of the U.S. AP examinee population
c. $df = 5$
d. chi-square distribution with $df = 5$
e. chi-square test statistic = 13.4

f. *p*-value = 0.0199

g. Check student's solution.

h. i. Alpha = 0.05

   ii. Decision: Reject null when *a* = 0.05

   iii. Reason for Decision: *p*-value < alpha

   iv. Conclusion: Local data do not fit the AP Examinee Distribution.

   v. Decision: Do not reject null when *a* = 0.01

   vi. Conclusion: There is insufficient evidence to conclude that local data do not follow the distribution of the U.S. AP examinee distribution.

77

a. H_0: The actual college majors of graduating females fit the distribution of their expected majors

b. H_a: The actual college majors of graduating females do not fit the distribution of their expected majors

c. *df* = 10

d. chi-square distribution with *df* = 10

e. test statistic = 11.48

f. *p*-value = 0.3211

g. Check student's solution.

h. i. Alpha = 0.05

   ii. Decision: Do not reject null when *a* = 0.05 and *a* = 0.01

   iii. Reason for decision: *p*-value > alpha

   iv. Conclusion: There is insufficient evidence to conclude that the distribution of actual college majors of graduating females fits the distribution of their expected majors.

79 true

81 true

83 false

85

a. H_0: Surveyed obese fit the distribution of expected obese

b. H_a: Surveyed obese do not fit the distribution of expected obese

c. *df* = 4

d. chi-square distribution with *df* = 4

e. test statistic = 54.01

f. *p*-value = 0

g. Check student's solution.

h. i. Alpha: 0.05

   ii. Decision: Reject the null hypothesis.

   iii. Reason for decision: *p*-value < alpha

   iv. Conclusion: At the 5% level of significance, from the data, there is sufficient evidence to conclude that the surveyed obese do not fit the distribution of expected obese.

87

a. H_0: Car size is independent of family size.

b. H_a: Car size is dependent on family size.

c. *df* = 9
d. chi-square distribution with *df* = 9
e. test statistic = 15.8284
f. *p*-value = 0.0706
g. Check student's solution.
h.
   i. Alpha: 0.05
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: *p*-value > alpha
   iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that car size and family size are dependent.

89

a. H_0: Honeymoon locations are independent of bride's age.
b. H_a: Honeymoon locations are dependent on bride's age.
c. *df* = 9
d. chi-square distribution with *df* = 9
e. test statistic = 15.7027
f. *p*-value = 0.0734
g. Check student's solution.
h.
   i. Alpha: 0.05
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: *p*-value > alpha
   iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that honeymoon location and bride age are dependent.

91

a. H_0: The types of fries sold are independent of the location.
b. H_a: The types of fries sold are dependent on the location.
c. *df* = 6
d. chi-square distribution with *df* = 6
e. test statistic =18.8369
f. *p*-value = 0.0044
g. Check student's solution.
h.
   i. Alpha: 0.05
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: *p*-value < alpha
   iv. Conclusion: At the 5% significance level, There is sufficient evidence that types of fries and location are dependent.

93

a. H_0: Salary is independent of level of education.
b. H_a: Salary is dependent on level of education.
c. *df* = 12
d. chi-square distribution with *df* = 12
e. test statistic = 255.7704

f. p-value = 0
g. Check student's solution.
h. Alpha: 0.05

   Decision: Reject the null hypothesis.

   Reason for decision: p-value < alpha

   Conclusion: At the 5% significance level, there is sufficient evidence to conclude that salary and level of education are dependent.

95 true

97 true

99

a. H_0: Age is independent of the youngest online entrepreneurs' net worth.
b. H_a: Age is dependent on the net worth of the youngest online entrepreneurs.
c. $df = 2$
d. chi-square distribution with $df = 2$
e. test statistic = 1.76
f. p-value 0.4144
g. Check student's solution.
h.
   i. Alpha: 0.05
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: p-value > alpha
   iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that age and net worth for the youngest online entrepreneurs are dependent.

101

a. H_0: The distribution for personality types is the same for both majors
b. H_a: The distribution for personality types is not the same for both majors
c. $df = 4$
d. chi-square with $df = 4$
e. test statistic = 3.01
f. p-value = 0.5568
g. Check student's solution.
h.
   i. Alpha: 0.05
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: p-value > alpha
   iv. Conclusion: There is insufficient evidence to conclude that the distribution of personality types is different for business and social science majors.

103

a. H_0: The distribution for fish caught is the same in Green Valley Lake and in Echo Lake.
b. H_a: The distribution for fish caught is not the same in Green Valley Lake and in Echo Lake.
c. 3
d. chi-square with $df = 3$
e. 11.75

f. p-value = 0.0083

g. Check student’s solution.

h. i. Alpha: 0.05

   ii. Decision: Reject the null hypothesis.

   iii. Reason for decision: p-value < alpha

   iv. Conclusion: There is evidence to conclude that the distribution of fish caught is different in Green Valley Lake and in Echo Lake

105

a. H_0: The distribution of average energy use in the USA is the same as in Europe between 2005 and 2010.

b. H_a: The distribution of average energy use in the USA is not the same as in Europe between 2005 and 2010.

c. $df = 4$

d. chi-square with $df = 4$

e. test statistic = 2.7434

f. p-value = 0.7395

g. Check student’s solution.

h. i. Alpha: 0.05

   ii. Decision: Do not reject the null hypothesis.

   iii. Reason for decision: p-value > alpha

   iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that the average energy use values in the US and EU are not derived from different distributions for the period from 2005 to 2010.

107

a. H_0: The distribution for technology use is the same for community college students and university students.

b. H_a: The distribution for technology use is not the same for community college students and university students.

c. 2

d. chi-square with $df = 2$

e. 7.05

f. p-value = 0.0294

g. Check student’s solution.

h. i. Alpha: 0.05

   ii. Decision: Reject the null hypothesis.

   iii. Reason for decision: p-value < alpha

   iv. Conclusion: There is sufficient evidence to conclude that the distribution of technology use for statistics homework is not the same for statistics students at community colleges and at universities.

110 225

112 H_0: $\sigma^2 \leq 150$

114 36

116 Check student’s solution.

118 The claim is that the variance is no more than 150 minutes.

120 a Student's t- or normal distribution

122

a. H_0: $\sigma = 15$
b. H_a: $\sigma > 15$
c. $df = 42$
d. chi-square with $df = 42$
e. test statistic = 26.88
f. p-value = 0.9663
g. Check student's solution.
h. i. Alpha = 0.05
   ii. Decision: Do not reject null hypothesis.
   iii. Reason for decision: p-value > alpha
   iv. Conclusion: There is insufficient evidence to conclude that the standard deviation is greater than 15.

124

a. H_0: $\sigma \leq 3$
b. H_a: $\sigma > 3$
c. $df = 17$
d. chi-square distribution with $df = 17$
e. test statistic = 28.73
f. p-value = 0.0371
g. Check student's solution.
h. i. Alpha: 0.05
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: p-value < alpha
   iv. Conclusion: There is sufficient evidence to conclude that the standard deviation is greater than three.

126

a. H_0: $\sigma = 2$
b. H_a: $\sigma \neq 2$
c. $df = 14$
d. chi-square distiribution with $df = 14$
e. chi-square test statistic = 5.2094
f. p-value = 0.0346
g. Check student's solution.
h. i. Alpha = 0.05
   ii. Decision: Reject the null hypothesis
   iii. Reason for decision: p-value < alpha
   iv. Conclusion: There is sufficient evidence to conclude that the standard deviation is different than 2.

128 The sample standard deviation is \$34.29. $H_0 : \sigma^2 = 25^2$
$H_a : \sigma^2 > 25^2$
$df = n - 1 = 7$.
test statistic: $x^2 = \quad x_7^2 = \dfrac{(n-1)s^2}{25^2} = \dfrac{(8-1)(34.29)^2}{25^2} = 13.169$;
p-value: $P\left(x_7^2 > 13.169\right) = 1 - P\left(x_7^2 \quad \leq 13.169\right) = 0.0681$
Alpha: 0.05

Decision: Do not reject the null hypothesis.
Reason for decision: *p*-value > alpha
Conclusion: At the 5% level, there is insufficient evidence to conclude that the variance is more than 625.

130

a. The test statistic is always positive and if the expected and observed values are not close together, the test statistic is large and the null hypothesis will be rejected.
b. Testing to see if the data fits the distribution “too well” or is too perfect.